

9/13/16

"Coming together is a beginning keeping together is progress working together is success."
-Henry Ford

HW: Test 1 on Tuesday 9/20

AIM: What does it mean for a function to be continuous?

Warm Up:

Fill in the 1st column of the chart on the packet.

Limits and Continuity

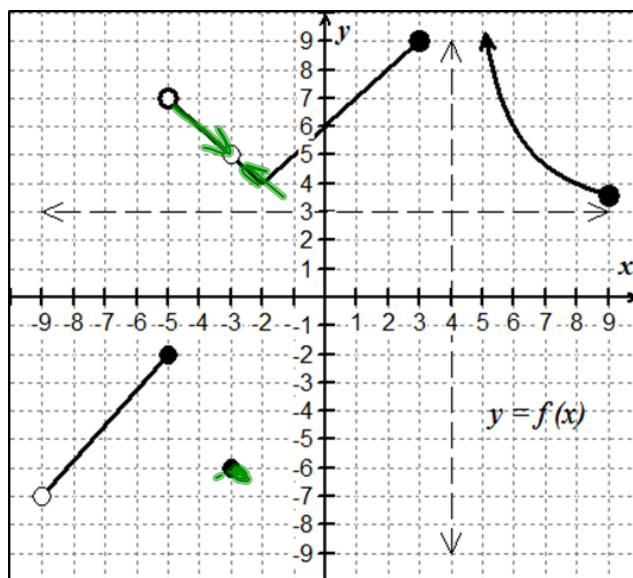


Use the graph below to complete the table with the given information. After filling in the table, you should be able to write the three pieces of information that must be true in order for a function, $F(x)$, to be continuous at $x = a$.

1. $f(a)$ is defined
(There is a point)

2. $\lim_{x \rightarrow a} f(x)$ exists
(There is a limit @ $x=a$)

3. $\lim_{x \rightarrow a} f(x) = f(a)$

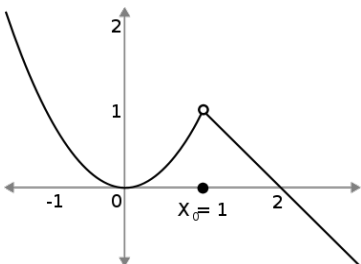
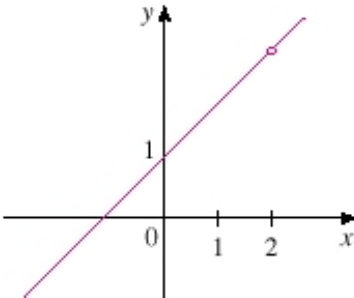
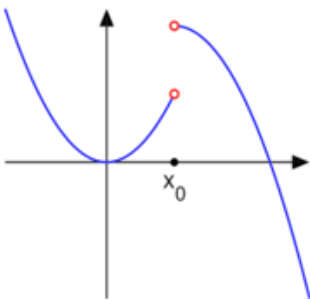
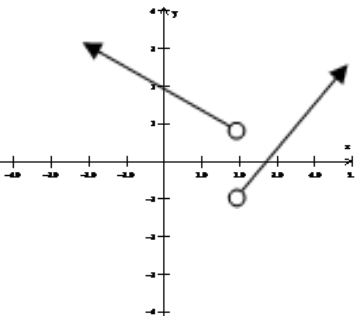
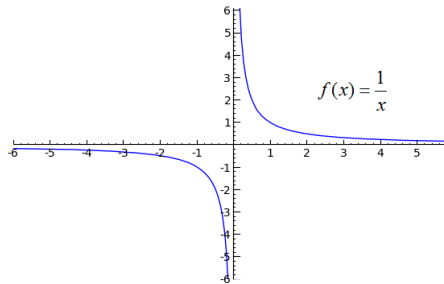
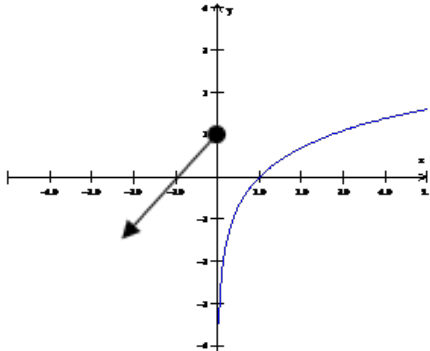




$x = a$	Is the function defined? If so, what is its value?	What is the value of $\lim_{x \rightarrow a^-} F(x)$?	What is the value of $\lim_{x \rightarrow a^+} F(x)$?	What is the value of $\lim_{x \rightarrow a} F(x)$?	Is $F(x)$ continuous at $x = a$?
$x = 9$	3.5	3.5	DNE	DNE	No
$x = 4$	DNE	DNE	∞	DNE	No
$x = 3$	9	9	DNE	DNE	No
$x = 0$	6	6	6	6	Yes
$x = -3$	-6	5	5	5	No
$x = -5$	-2	-2	7	DNE	No

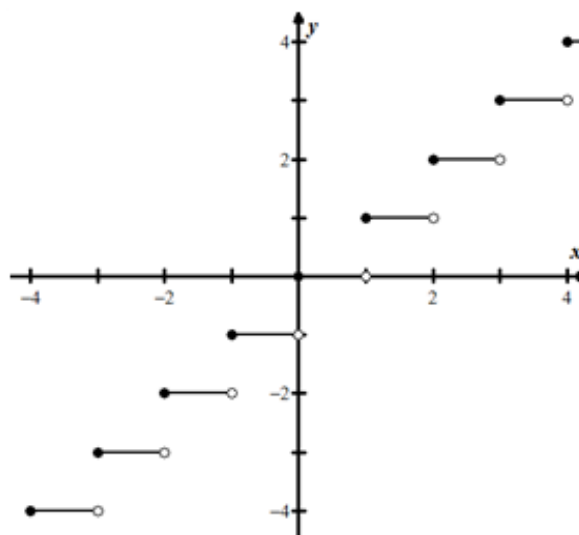
While **limits** told us where a function intended to go.
Continuity guarantees that the function actually made it there.

Classifying Discontinuities

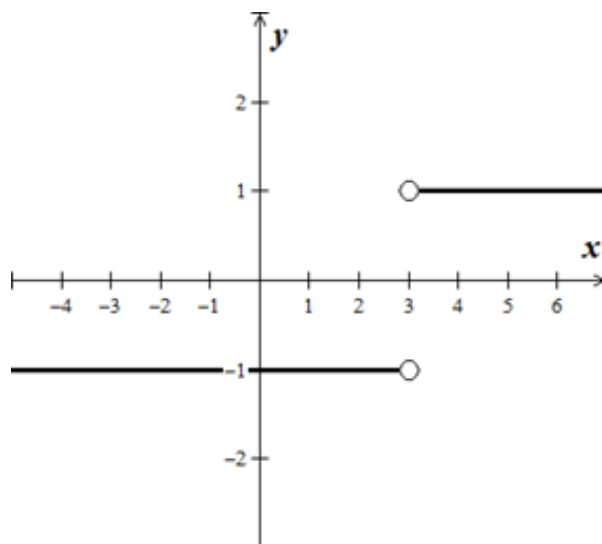
Removable or Point (Holes)	Non-Removable	
2-sided limit exists	Jump 1-sided limits exists	Infinite At least one of the 1-sided limits doesn't exist.
 	 	 

EX #1: Discuss the continuity of the following:

A. $f(x) = [x]$



B. $g(x) = \frac{|x-3|}{x-3}$



Definition of Continuity – More Facts and Theorems

Continuity on an Open Interval:

A function is continuous on an open interval (a, b) if it is continuous at each point in the interval.

Continuity on a Closed Interval:

A function is continuous on a closed interval $[a, b]$ if it is continuous on the open interval (a, b) and the function is continuous from the right at a and continuous from the left at b .

Properties of Continuity:

Given functions f and g continuous at $x = c$, then the following functions are also continuous at $x = c$.

1. Scalar multiple: $b \cdot f$
2. Sum or difference: $f \pm g$
3. Product: $f \cdot g$
4. Quotient: $\frac{f}{g}$; if $g(c) \neq 0$
5. Compositions: If g is continuous at c and f is continuous at $g(c)$, then the composite function is continuous at c , $(f \circ g)(x) = f(g(x))$

Continuity Laws of Some Basic Functions:

- Polynomial functions $P(x)$ are continuous over reals
- Rational Functions $P(x)/Q(x)$ is continuous on its domain such that $Q(c) \neq 0$
- $y = x^{1/n}$ is continuous on all reals if n is odd and continuous on $[0, \infty)$ if n is even.
- $y = \sin x$ and $y = \cos x$
- $y = b^x$ is continuous for $b > 0, b \neq 1$
- $y = \log_b x$ is continuous for $x > 0, b > 0, b \neq 1$

Inverse functions - if $f(x)$ is continuous on an interval with range R and $f^{-1}(x)$ exists, then $f^{-1}(x)$ is continuous on domain R .

One-Sided Continuity: A function $f(x)$ is called:

Left-continuous at $x = c$ if $\lim_{x \rightarrow c^-} f(x) = f(c)$

Right-continuous at $x = c$ if $\lim_{x \rightarrow c^+} f(x) = f(c)$

Continuity at a Point:

Suppose $f(x)$ is defined on an open interval containing $x = c$. Then $f(x)$ is continuous at $x = c$ if

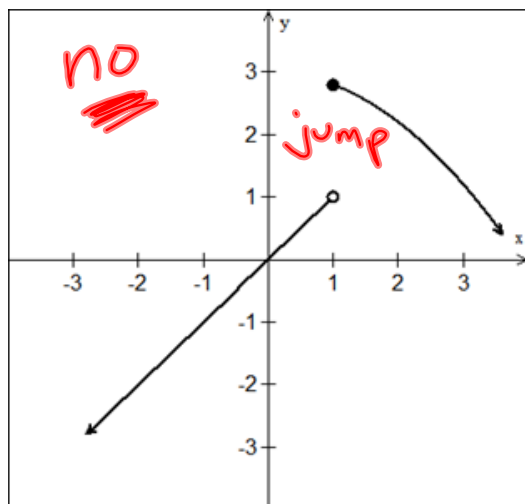
$$\lim_{x \rightarrow c} f(x) = f(c)$$

Definition of Continuity

A function f is continuous at c if the following three conditions are met:

1. $f(c)$ is defined.
2. $\lim_{x \rightarrow c} f(x)$ exists.
3. $\lim_{x \rightarrow c} f(x) = f(c)$

EX #2: Continuous or Not? Defend your answer with a condition from the three point definition of continuity.



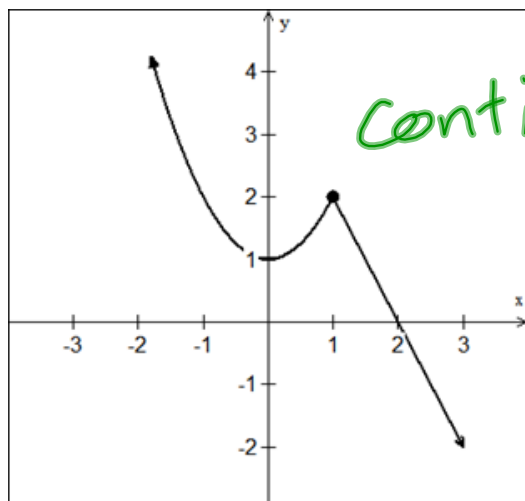
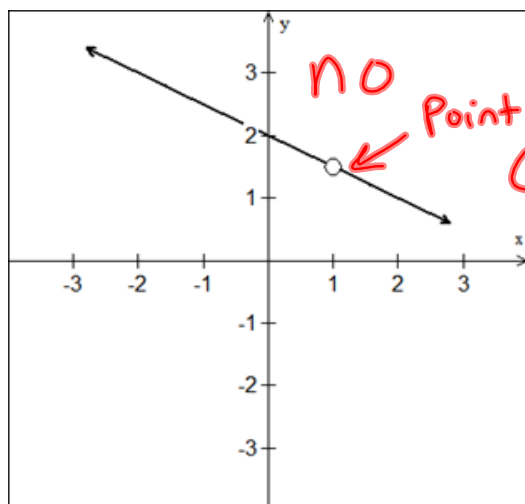
$$1) f(1) = 3$$

$$2) \lim_{x \rightarrow 1^-} f(x) = 1$$

$$\lim_{x \rightarrow 1^+} f(x) = 3$$

$$\lim_{x \rightarrow 1} f(x) = \text{DNE}$$

$$3) \lim_{x \rightarrow 1} f(x) \neq f(1)$$



EX #3: Without graphing, justify why the functions are continuous for all real numbers.

A. $y = x^{\frac{2}{3}}$ ← power
← root

nth root is odd
∴ it is continuous

B. $y = e^x$

exponential functions
are continuous

C. $f(x) = 2x - \cos x$

line
(continuous)

trig
(continuous)

∴ $f(x)$ is continuous

D. $g(x) = \frac{1}{x^2 + 1}$

$x^2 + 1 \neq 0$

Continuous everywhere

EX #4: If the function f is continuous and if $f(x) = \frac{x^2 - 4}{x + 2}$ when $x \neq -2$, then $f(-2) = ?$

EX #5: For what value(s) of the constant c is the function g continuous over all the reals?

$$g(x) = \begin{cases} cx + 1 & \text{if } x \leq 3 \\ cx^2 - 1 & \text{if } x > 3 \end{cases}$$

Want these
to meet
@ $x = 3$

Plug in $x = 3$ and then
set pieces equal \rightarrow quadratic

$$c(3) + 1 = c(3)^2 - 1$$

$$\begin{array}{r} 3c + 1 = 9c - 1 \\ -3c + 1 \quad -3c + 1 \\ \hline \end{array}$$

$$2 = 6c$$

$$c = \frac{1}{3}$$

EX #6: Given $h(x) = \begin{cases} \text{line } -2x-5; & x < -2 \\ \underline{3}; & x = -2 \leftarrow \text{point} \\ \text{cubic } x^3 - 6x + 3; & x > -2 \leftarrow \text{cubic} \end{cases}$

for what values of x is h not continuous? Justify.

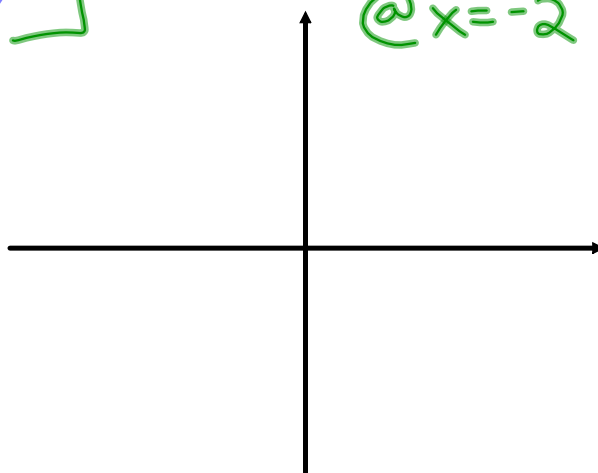
plug in $x = -2$

line $-2(-2) - 5 = -1$

Point $3 = 3$

cubic $(-2)^3 - 6(-2) + 3 = 7$

All different
 $h(x)$ is NOT continuous
 @ $x = -2$



HW: Limits and Continuity HW #1-14
Test Tuesday 9/20